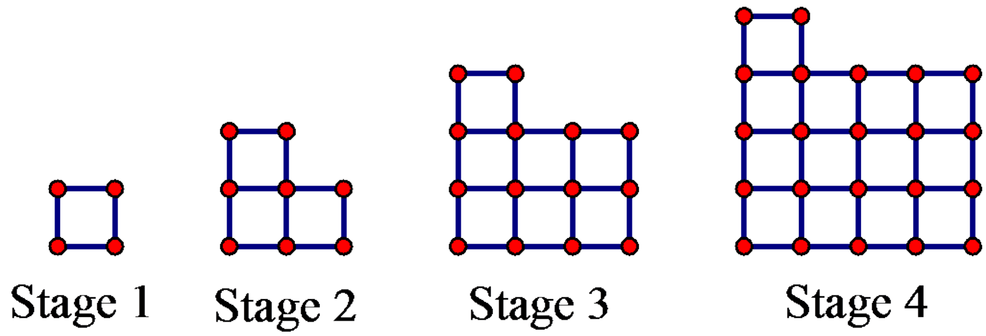


1. How many 1 x 1 squares are in each stage of this pattern?

2. What might stage 5 of this pattern look like? How many 1 x 1 squares would be in stage 5?

3. Write an expression that describes the number of 1 x 1 squares in stage *n* of the pattern. Justify your answer geometrically by referring to the pattern.

4. How much does the number of squares change from stage 1 to stage 2 of the pattern?

5. How much does the number of squares change from stage 2 to stage 3 of the pattern?

6. How much does the number of squares change from stage 3 to stage 4 of the pattern?

7. What do your answers to 4-6 tell you about the rate of change of the number of squares with respect to the stage number?

8. You have previously worked with linear and exponential functions. Can this pattern be expressed as a linear or exponential function? Why or why not?

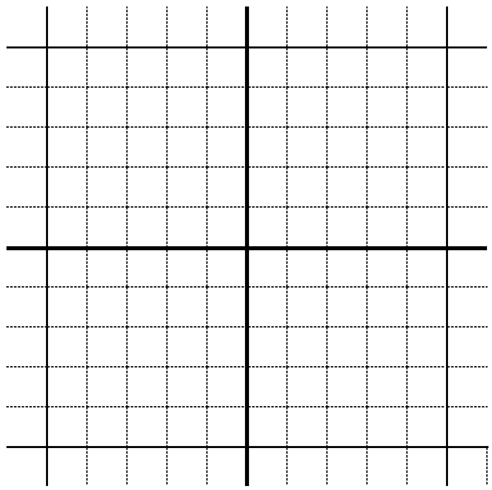
**Graphing Transformations

Adapted from Marilyn Munford, Fayette County School System

You will graph various functions and make conjectures based on the patterns you observe from the original function $y=x^2$
Follow the directions below and answer the questions that follow.

- Fill in the t-chart and sketch the parent graph $\mathbf{y = x^2}$ below.

x	$\mathbf{y = x^2}$
-3	
-2	
-1	
0	
1	
2	
3	



- Now, for each set of problems below, describe what happened to the graph ($y_1 = \mathbf{x^2}$) to get the new functions.

Equation	Changes to parent graph.
$y_1 = \mathbf{x^2}$ $y_2 = \mathbf{x^2} +3$ $y_3 = \mathbf{x^2} +7$	

1. Conjecture: The graph of $y = x^2 + a$ will cause the parent graph to __________.

Equation	Changes to parent graph.
$y_1 = \mathbf{x^2}$ $y_2 = \mathbf{x^2} -3$ $y_3 = \mathbf{x^2} -7$	

2. Conjecture: The graph of $y = x^2 - a$ will cause the parent graph to __________.

Equation	Changes to parent graph.
$y_1 = \mathbf{x^2}$ $y_2 = \mathbf{(x+3)^2}$ $y_3 = \mathbf{(x+7)^2}$	

3. Conjecture: The graph of $y = (x + a)^2$ will cause the parent graph to __________.

Equation	Changes to parent graph.
$y_1 = \mathbf{x^2}$ $y_2 = \mathbf{(x-3)^2}$ $y_3 = \mathbf{(x-3)^2}$	

4. Conjecture: The graph of $y = (x - a)^2$ will cause the parent graph to __________.

Equation	Changes to parent graph.
$y_1 = \mathbf{x}^2$ $y_2 = -\mathbf{x}^2$ $y_3 = -3\mathbf{x}^2$	

5. Conjecture: Multiplying the parent graph by a negative causes the parent graph to __________.

For the following graphs, please use the descriptions "vertical stretch" (skinny) or "vertical shrink" (fat).

Equation	Changes to parent graph.
$y_1 = \mathbf{x}^2$ $y_2 = 3\mathbf{x}^2$ $y_3 = 7\mathbf{x}^2$	

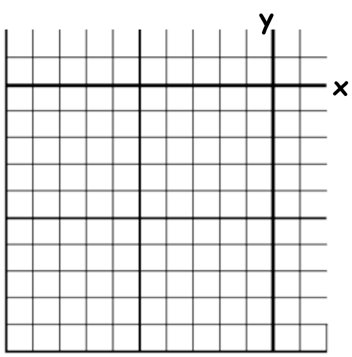
6. Conjecture: Multiplying the parent graph by a number whose absolute value is greater than one causes the parent graph to_______________.

Equation	Changes to parent graph.
$y_1 = \mathbf{x}^2$ $y_2 = \frac{1}{2}\,\mathbf{x}^2$ $y_3 = \frac{1}{4}\,\mathbf{x}^2$	

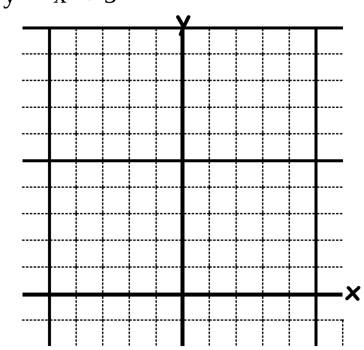
7. Conjecture: Multiplying the parent graph by a number whose absolute value is between zero and one causes the parent graph to __________.

Based on your conjectures above, sketch the graphs <u>without</u> using your graphing calculator.

8. $y = (x+3)^2 - 4$



9. $y = -x^2 + 5$



Now, go back and graph these on your graphing calculator and see if you were correct. Were you?

Based on your conjectures, write the equations for the following transformations to $y=x^2$.

10. Translated 6 units up

11. Translated 2 units right

12. Stretched vertically by a factor of 3

13. Reflected over the x-axis, 2 units left and down 5 units

**Paula's Peaches

Paula is a peach grower in central Georgia and wants to expand her peach orchard. In her current orchard, there are 30 trees per acre and the average yield per tree is 600 peaches. Data from the local agricultural experiment station indicates that if Paula plants more than 30 trees per acre, once the trees are in production, the average yield of 600 peaches per tree will decrease by 12 peaches for each tree over 30. She needs to decide how many trees to plant in the new section of the orchard.

1. Paula believes that algebra can help her determine the best plan for the new section of orchard and begins by developing a mathematical model of the relationship between the number of trees per acre and the average yield in **peaches per tree.**
   a. Is this relationship linear or nonlinear? Explain your reasoning.

   b. If Paula plants 6 more trees per acre, what will be the **average** yield in peaches per tree? What is the **average** yield in peaches per tree if she plants 42 trees per acre?

   c. Let T be the function for which the input x is the number of trees planted on each acre and $T(x)$ is the average yield in peaches per tree. Write a formula for $T(x)$ in terms of x and express it in simplest form. Explain how you know that your formula is correct.

   d. Draw a graph of the function T. Given that the information from the agricultural experiment station applies only to increasing the number of trees per acre, what is an appropriate domain for the function T?

2. Since her income from peaches depends on the total number of peaches she produces, Paula realized that she needed to take a next step and consider the total number of peaches that she can produce **per acre**.

   a. With the current 30 trees per acre, what is the yield in total peaches per acre? If Paula plants 36 trees per acre, what will be the yield in total peaches per acre? 42 trees per acre?

   b. Find the average rate of change of peaches per acre with respect to number of trees per acre when the number of trees per acre increases from 30 to 36. Write a sentence to explain what this number means.

   c. Find the average rate of change of peaches per acre with respect to the number of trees per acre when the number of trees per acre increases from 36 to 42. Write a sentence to explain the meaning of this number.

   d. Is the relationship between number of trees per acre and yield in peaches per acre linear? Explain your reasoning.

e. Let Y be the function that expresses this relationship; that is, the function for which the input x is the number of trees planted on each acre and the output $Y(x)$ is the total yield in peaches per acre. Write a formula for $Y(x)$ in terms of x and express your answer in expanded form.

f. Calculate $Y(30)$, $Y(36)$, and $Y(42)$. What is the meaning of these values? How are they related to your answers to parts a through c?

g. What is the relationship between the domain for the function T and the domain for the function Y? Explain.

3. Paula wants to know whether there is a different number of trees per acre that will give the same yield per acre as the yield when she plants 30 trees per acre.

a. Write an equation that expresses the requirement that x trees per acre yields the same total number of peaches per acre as planting 30 trees per acre.

b. Use the algebraic rules for creating equivalent equations to obtain an equivalent equation with an expression in x on one side of the equation and 0 on the other.

c. Multiply this equation by an appropriate rational number so that the new equation is of the form $x^2 + bx + c = 0$. Explain why this new equation has the same solution set as the equations from parts a and b.

d. When the equation is in the form $x^2 + bx + c = 0$, what are the values of b and c

e. Find integers m and n such that $m \cdot n = c$ and $m + n = b$.

f. Using the values of m and n found in part e, form the algebraic expression $(x+m)(x+n)$ and simplify it.

g. Combining parts d through f, rewrite the equation from part c in the form $(x+m)(x+n) = 0$.

h. This equation expresses the idea that the product of two numbers, $x + m$ and $x + n$, is equal to 0. We know from the discussion in Unit 2 that, when the product of two numbers is 0, one of the numbers has to be 0. This property is called the ***Zero Product Property***. For these particular values of m and n, what value of x makes $x + m = 0$ and what value of x makes $x + n = 0$?

i. Verify that the answers to part h are solutions to the equation written in part a. It is appropriate to use a calculator for the arithmetic.

j. Write a sentence to explain the meaning of your solutions to the equation in relation to planting peach trees.

4. Paula saw another peach grower, Sam, from a neighboring county at a farm equipment auction and began talking to him about the possibilities for the new section of her orchard. Sam was surprised to learn about the agricultural research and said that it probably explained the drop in yield for a orchard near him. This peach farm has more than 30 trees per acre and is getting an average total yield of 14,400 peaches per acre. *(Remember: Throughout this task assume that, for all peach growers in this area, the average yield is 600 peaches per tree when 30 trees per acre are planted and that this yield will decrease by 12 peaches per tree for each additional tree per acre.)*

   a. Write an equation that expresses the situation that x trees per acre results in a total yield per acre of 14,400 peaches per acre.

   b. Use the algebraic rules for creating equivalent equations to obtain an equivalent equation with an expression in x on one side of the equation and 0 on the other.

   c. Multiply this equation by an appropriate rational number so that the new equation is of the form $x^2 + bx + c = 0$. Explain why this new equation has the same solution set as the equations from parts and b.

   d. When the equation is in the form $x^2 + bx + c = 0$, what is value of b and what is the value of c?

   e. Find integers m and n such that $m \cdot n = c$ and $m + n = b$.

   f. Using the values of m and n found in part e, form the algebraic expression $(x+m)(x+n)$ and simplify it.

   g. Combining parts d through f, rewrite the equation from part d in the form $(x+m)(x+n)=0$.

   h. This equation expresses the idea that the product of two numbers, $x+m$ and $x+n$, is equal to 0. We know from the discussion in Unit 2 that, when the product of two numbers is 0, one of the numbers has to be 0. What value of x makes $x+m=0$? What value of x makes $x+n=0$?

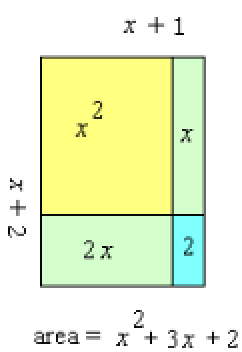
   i. Verify that the answers to part h are solutions to the equation written in part a. It is appropriate to use a calculator for the arithmetic.

   j. Which of the solutions verified in part i is (are) in the domain of the function Y? How many peach trees per acre are planted at the peach orchard getting 14400 peaches per acre?

The steps in items 3 and 4 outline a method of solving equations of the form $x^2 + bx + c = 0$. These equations are called ***quadratic equations*** and an expression of the form $x^2 + bx + c$ is called a ***quadratic expression***. In general, quadratic expressions may have any nonzero coefficient on the x^2 term. An important part of this method for solving quadratic equations is the process of rewriting an expression of the form $x^2 + bx + c$ in the form $(x+m)(x+n)$. The identity tells us that the product of the numbers *m* and *n* must equal *c* and that the sum of *m* and *n* must equal *b*.

5. Since the whole expression $(x+m)(x+n)$ is a product, we call the expressions $x+m$ and $x+n$ the **factors** of this product. For the following expressions in the form $x^2 + bx + c$, rewrite the expression as a product of factors of the form *x* + *m* and *x* + *n*. Verify each answer by drawing a rectangle with sides of length $x+m$ and $x+n$, respectively, and showing geometrically that the area of the rectangle is $x^2 + bx + c$.

Example: $x^2 + 3x + 2$
Solution: (x + 1)*(x + 2)



a. $x^2 + 6x + 5$

b. $x^2 + 5x + 6$

c. $x^2 + 7x + 12$

d. $x^2 + 8x + 12$

On a separate sheet of paper:

e. $x^2 + 13x + 36$

f. $x^2 + 13x + 12$

6. In item 5, the values of b and c were positive. Now use Identity 1 in reverse to factor each of the following quadratic expressions of the form $x^2 + bx + c$ where c is positive but b is negative. Verify each answer by multiplying the factored form to obtain the original expression.

On a separate sheet of paper:

a. $x^2 - 8x + 7$
b. $x^2 - 9x + 18$
c. $x^2 - 4x + 4$
d. $x^2 - 8x + 15$
e. $x^2 - 11x + 24$
f. $x^2 - 11x + 18$
g. $x^2 - 12x + 27$

Paula's Peaches Continued!

7. Now we return to the peach growers in central Georgia. How many peach trees per acre would result in only 8400 peaches per acre? Which answer makes sense in this particular context?

8. If there are no peach trees on a property, then the yield is zero peaches per acre. Write an equation to express the idea that the yield is zero peaches per acre with x trees planted per acre, where x is number greater than 30. Is there a solution to this equation? Explain why only one of the solutions makes sense in this context.

9. At the same auction where Paula heard about the peach grower who was getting a low yield, she talked to the owner of a major farm supply store in the area. Paula began telling the store owner about her plans to expand her orchard, and the store owner responded by telling her about a local grower that gets 19,200 peaches per acre. Is this number of peaches per acre possible? If so, how many trees were planted?

10. Using graph paper, explore the graph of Y as a function of x.
    a. What points on the graph correspond to the answers for part j from questions 3 and 4?

    b. What points on the graph correspond to the answers to questions 7, 8, and 9?

    c. What is the relationship of the graph of the function Y to the graph of the function f , where the formula for $f(x)$ is the same as the formula for $Y(x)$ but the domain for f is all real numbers?

    d. Questions 4, 7, and 8 give information about points that are on the graph of f but not on the graph of Y. What points are these?

    e. Graph the functions f and Y on the same axes. How does your graph show that the domain of f is all real numbers? How is the domain of Y shown on your graph?

    f. Draw the line $y = 18000$ on the graph drawn for item 10, part e. This line is the graph of the function with constant value 18000. Where does this line intersect the graph of the function Y? Based on the graph, how many trees per acre give a yield of more than 18000 peaches per acre?

    g. Draw the line $y = 8400$ on your graph. Where does this line intersect the graph of the function Y? Based on the graph, how many trees per acre give a yield of <u>fewer than</u> 8400 peaches per acre?

    h. Use a graphing utility and this intersection method to find the number of trees per acre that give a total yield **closest** to the following numbers of peaches per acre:
    (i) 10000 (ii) 15000 (iii) 20000

i. Find the value of the function Y for the number of trees given in answering (i) – (iii) in part c above.

13. In items 5 and 6, we used factoring as part of a process to solve equations that are equivalent to equations of the form $x^2 + bx + c = 0$ where b and c are integers. Look back at the steps you did in items 3 and 4, and describe the process for solving an equation of the form $x^2 + bx + c = 0$. Use this process to solve each of the following equations, that is, to find all of the numbers that satisfy the original equation. Verify your work by checking each solution in the original equation.

a. $x^2 - 6x + 8 = 0$

b. $x^2 - 15x + 36 = 0$

c. $x^2 + 28x + 27 = 0$

d. $x^2 - 3x - 10 = 0$

e. $x^2 + 2x - 15 = 0$

f. $x^2 - 4x - 21 = 0$

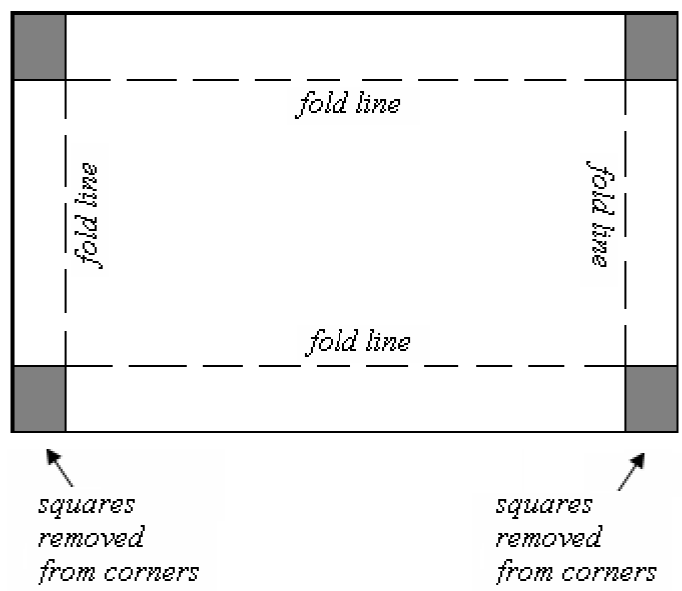
g. $x^2 - 7x = 0$

h. $x^2 + 13x = 0$

11. For each of the equations solved in question 11, do the following.
    a. Use technology to graph a function whose formula is given by the left-hand side of the equation.
    b. Find the points on the graph which correspond to the solutions found in question 8.
    c. How is each of these results an example of the intersection method explored above?

****Henley's Chocolates**

Henley Chocolates is famous for its mini chocolate truffles, which are packaged in foil covered boxes. The base of each box is created by cutting squares that are 4 centimeters on an edge from each corner of a rectangular piece of cardboard and folding the cardboard edges up to create a rectangular prism 4 centimeters deep. A matching lid is constructed in a similar manner, but, for this task, we focus on the base, which is illustrated in the diagrams below.

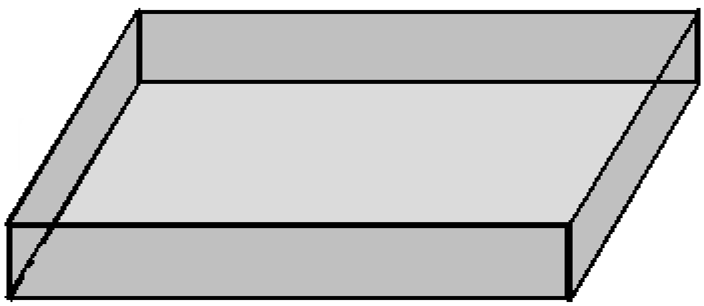


For the base of the truffle box, paper tape is used to join the cut edges at each corner. Then the inside and outside of the truffle box base are covered in foil.

Henley Chocolates sells to a variety retailers and creates specific box sizes in response to requests from particular clients. However, Henley Chocolates requires that their truffle boxes always be 4 cm deep and that, in order to preserve the distinctive shape associated with Henley

Chocolates, the bottom of each truffle box be a rectangle that is two and one-half times as long as it is wide.

1. Henley Chocolates restricts box sizes to those which will hold plastic trays for a whole number of mini truffles. A box needs to be at least 2 centimeters wide to hold one row of mini truffles. Let L denote the length of a piece of cardboard from which a truffle box is made. What value of L corresponds to a finished box base for which the bottom is a rectangle that is 2 centimeters wide?



base of the truffle box

2. Henley Chocolates has a maximum size box of mini truffles that it will produce for retail sale. For this box, the bottom of the truffle box base is a rectangle that is 50 centimeters long. What are the dimensions of the piece of cardboard from which this size truffle box base is made?

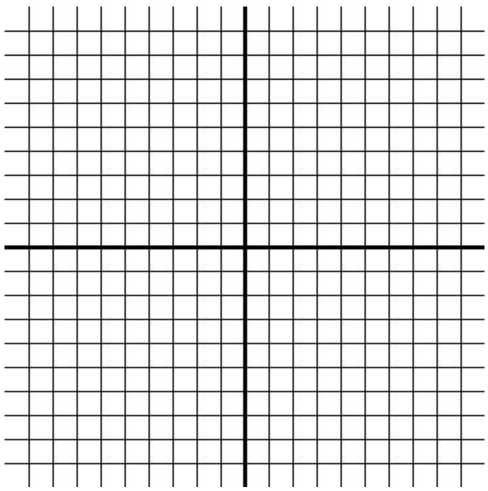
3. Since all the mini truffle boxes are 4 centimeters deep, each box holds two layers of mini truffles. Thus, the number of truffles that can be packaged in a box depends the number of truffles that can be in one layer, and, hence, on the area of the bottom of the box. Let $A(x)$ denote the area, in square centimeters, of the rectangular bottom of a truffle box base. Write a formula for $A(x)$ in terms of the length L, in centimeters, of the piece of cardboard from which the truffle box base is constructed.

4. Although Henley Chocolates restricts truffle box sizes to those that fit the plastic trays for a whole number of mini truffles, the engineers responsible for box design find it simpler to study the function A on the domain of all real number values of L in the interval from the minimum value of L found in item 1 to the maximum value of L found in item 2. State this interval of L values as studied by the engineers at Henley Chocolates.

5. Let g be the function with the same formula as the formula for function A but with domain all real numbers. Describe the transformations of the function f, the square function, that will

produce the graph of the function *g*. Use **technology to graph** *f* and *g* on the same axes to check that the graphs match your description of the described transformations.

6. Describe the graph of the function *A* in words and make a hand drawn sketch. Remember that you found the domain of the function in item 4. What is the range of the function *A*?



7. The engineers at Henley Chocolates responsible for box design have decided on two new box sizes that they will introduce for the next winter holiday season.
   a. The area of the bottom of the larger of the new boxes will be 640 square centimeters. Use the function *A* to write and solve an equation to find the length *L* of the cardboard need to make this new box.

   b. The area of the bottom of the smaller of the new boxes will be 40 square centimeters. Use the function *A* to write and solve an equation to find the length *L* of the cardboard need to make this new box.

8. How many mini-truffles do you think the engineers plan to put in each of the new boxes?

*Completing the Square & Deriving the Quadratic Formula (Spotlight Task)

Standards Addressed in this Task
MCC9-12.A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

MCC9-12.A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.★

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)

Standards for Mathematical Practice
7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

Section 1: Area models for multiplication

1. If the sides of a rectangle have lengths $x + 3$ and $x + 5$, what is an expression for the area of the rectangle? Draw the rectangle, label its sides, and indicate each part of the area.

2. For each of the following, draw a rectangle with side lengths corresponding to the factors given. Label the sides and the area of the rectangle:
   a. $(x + 3)(x + 4)$

   b. $(x + 1)(x + 7)$

c. $(x - 2)(x + 5)$

d. $(2x + 1)(x + 3)$

Section 2: Factoring by thinking about area and linear quantities
For each of the following, draw a rectangle with the indicated area. Find appropriate factors to label the sides of the rectangle.

1. $x^2 + 3x + 2$

2. $x^2 + 5x + 4$

3. $x^2 + 7x + 6$

4. $x^2 + 5x + 6$

5. $x^2 + 6x + 8$

6. $x^2 + 8x + 12$

7. $x^2 + 7x + 12$

8. $x^2 + 6x + 9$

9. $x^2 + 4x + 4$

Section 3: Completing the square

1. What number can you fill in the following blank so that $x^2 + 6x +$ ____ will have two equal factors? What are the factors? Draw the area and label the sides. What shape do you have?

2. What number can you fill in the following blank so that $x^2 + 8x +$ ____ will have two equal factors? What are the factors? Draw the area and label the sides. What shape do you have?

3. What number can you fill in the following blank so that $x^2 + 4x +$ ____ will have two equal factors? What are the factors? Draw the area and label the sides. What shape do you have?

4. What would you have to add to $x^2 + 10x$ in order to make a square? What could you add to $x^2 + 20x$ to make a square? What about $x^2 + 50x$? What if you had $x^2 + \boldsymbol{b}x$?

Section 4: Solving equations by completing the square

1. Solve $x^2 = 9$ without factoring. How many solutions do you have? What are your solutions?

2. Use the same method as in question 5 to solve $(x + 1)^2 = 9$. How many solutions do you have? What are your solutions?

3. In general, we can solve any equation of this form $(x + h)^2 = k$ by taking the square root of both sides and then solving the two equations that we get. Solve each of the following:
    a. $(x + 3)^2 = 16$

    b. $(x + 2)^2 = 5$

    c. $(x – 3)^2 = 4$

d. $(x - 4)^2 = 3$

4. Now, if we notice that we have the right combination of numbers, we can actually solve other equations by first putting them into this, using what we noticed in questions 1 – 4. Notice that if we have $x^2 + 6x + 9 = 25$, the left side is a square, that is, $x^2 + 6x + 9 = (x + 3)^2$. So, we can rewrite $x^2 + 6x + 9 = 25$ as $(x + 3)^2 = 25$, and then solve it just like we did the problems in question 7. (What do you get?)

5. Sometimes, though, the problem is not written quite in the right form. That's okay. We can apply what we already know about solving equations to write it in the right form, and then we can solve it. This is called completing the square. Let's say we have $x^2 + 6x = 7$. The left side of this equation is not a square, but we know what to add to it. If we add 9 to both sides of the equation, we get $x^2 + 6x + 9 = 16$. Now we can solve it just like the ones above. What is the solution?

6. Try these:
   a. $x^2 + 10x = -9$

   b. $x^2 + 8x = 20$

   c. $x^2 + 2x = 5$

   d. $x^2 + 6x - 7 = 0$

e. $2x^2 + 8x = -6$

Section 5: Deriving the quadratic formula by completing the square

If you can complete the square for a general quadratic equation, you will derive a formula you can use to solve any quadratic equation. Start with $ax^2 + bx + c = 0$, and follow the steps you used in Section 4.

*Standard to Vertex Form (Spotlight Task)

Standards Addressed in Task
MCC9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)
Write expressions in equivalent forms to solve problems
MCC9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)
MCC9-12.A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.★
MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)
MCC9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.★
MCC9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)
MCC9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. Reason abstractly and quantitatively by requiring students to make sense of quantities and their relationships to one another in problem situations.
3. Construct viable arguments and critique the reasoning of others by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

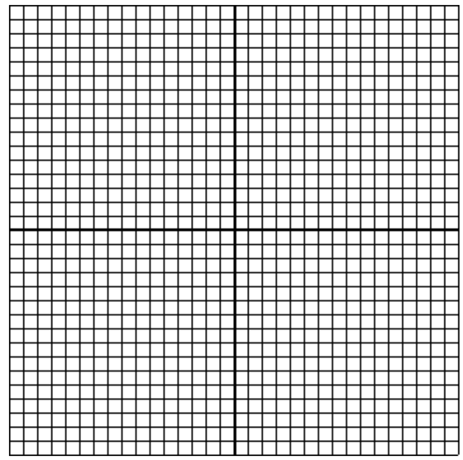
In this task you will learn to identify key features of quadratic functions by completing the square

In addition to solving equations, completing the square can be helpful in identifying horizontal and vertical shifts in the graph of a function. For instance, suppose you want to graph $f(x) = x^2 +$

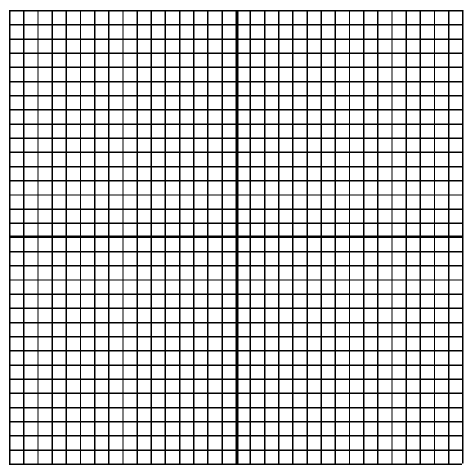
$6x + 5$. We can complete the square to write it in vertex form, so we want it to look like $f(x) = a(x - h)^2 + k$. We complete the square to find the $(x - h)^2$ part, and in doing so, we also find k. Look at $x^2 + 6x$. We know from our work earlier that we can add 9 to make a perfect square trinomial. But we can't just add 9 to an equation. We can add 9 and subtract 9 (because then we're just adding zero). So, we have $f(x) = (x^2 + 6x + 9) + 5 - 9$. When we simplify, we get $f(x) = (x + 3)^2 - 4$. So the graph of this function will be shifted three to the left and four down.

Find the horizontal and vertical shifts by completing the square and graph each of these:

5. $f(x) = x^2 + 10x + 27$



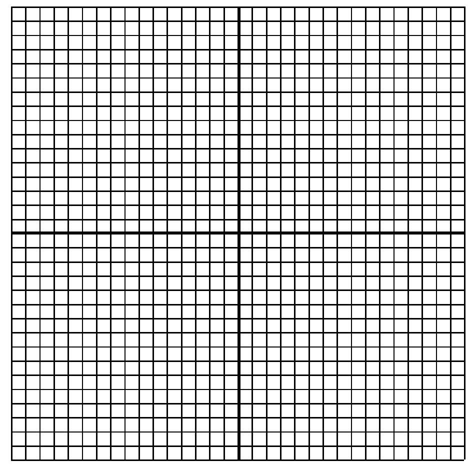
6. $f(x) = x^2 - 6x + 1$



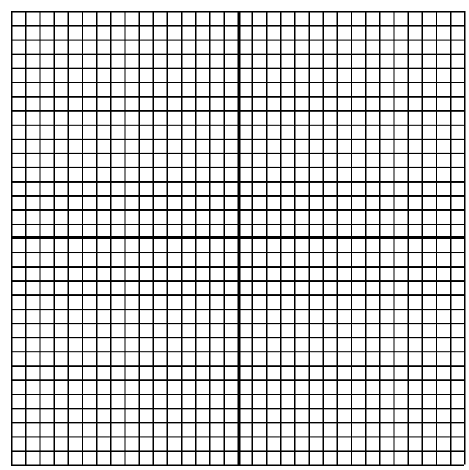
When the leading coefficient is not 1, we have to be even more careful when changing from standard to vertex form. However, the ideas are the same. We want to create a perfect square trinomial and write the equation in vertex form. For example, say we have $f(x) = 3x^2 + 6x + 5$. This time, we need to factor the leading coefficient out of the first two terms and then complete the square. So, we have $f(x) = 3(x^2 + 2x\ \ ) + 5$. Completing the square on $x^2 + 2x$ means we need to add 1. But if we add 1 inside the parentheses, we are actually adding three $(3 \cdot 1)$, so we have to add 3 and subtract 3: $f(x) = 3(x^2 + 2x + 1) + 5 - 3$. Simplifying, we have $f(x) = 3(x + 1)^2 + 2$.

Find the horizontal and vertical shifts by completing the square and graph each of these:

7. $f(x) = 2x^2 - 8x + 3$



8. $f(x) = -3x^2 + 12x - 5$

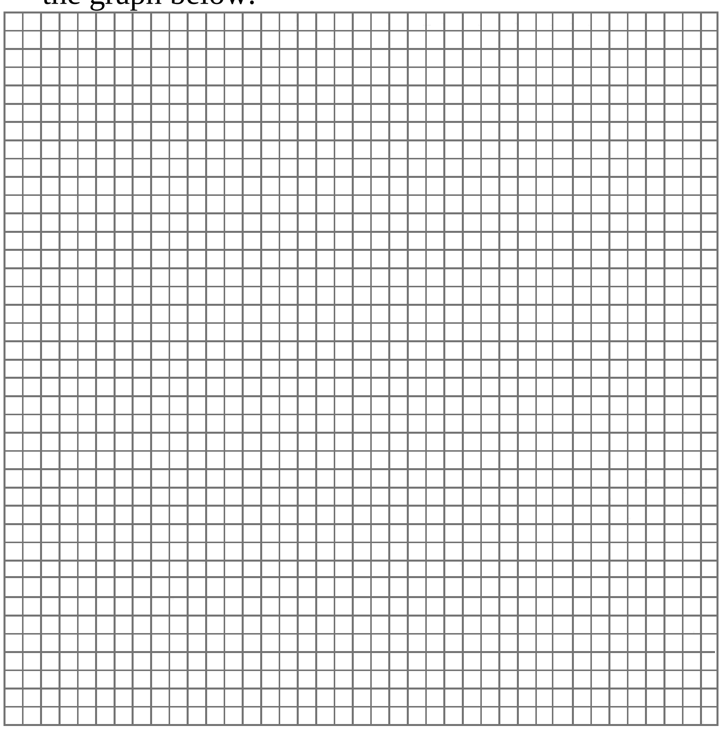


PROTEIN BAR TOSS

Blake and Zoe were hiking in a wilderness area. They came up to a scenic view at the edge of a cliff. As they stood enjoying the view, Zoe asked Blake if he still had some protein bars left, and, if so, could she have one. Blake said, "Here's one; catch!" As he said this, he pulled a protein bar out of his backpack and threw it up to toss it to Zoe. But the bar slipped out of his hand sooner than he intended, and the bar went straight up in the air with his arm out over the edge of the cliff. The protein bar left Blake's hand moving straight up at a speed of 24 feet per second. If we let t represent the number of seconds since the protein bar left the Blake's hand and let $h(t)$ denote the height of the bar, in feet above the ground at the base of the cliff, then, assuming that we can ignore the air resistance, we have the following formula expressing $h(t)$ as a function of t,

$$h(t) = -16t^2 + 24t + 160.$$

1. **Use technology to graph the equation** $y = -16t^2 + 24t + 160$. Remember t represents time and y represents height. Find a viewing window that includes the part of this graph that corresponds to the situation with Blake and his toss of the protein bar. What viewing window did you select? **Sketch** the graph below.



2. What was the height of the protein bar, measured from the ground at the base of the cliff, at the instant that it left Blake's hand? What special point on the graph is associated with this information?

3. At what other time does the protein bar reach the height from question 2? Describe how you reached your answer.

4. If Blake does not catch the falling protein bar, how long will it take for the protein bar to hit the base of the cliff? Justify your answer graphically. Then write a quadratic equation that you would need

to solve to justify the answer algebraically (*Note: you do not need to solve this equation at this point*).

The equation from item 5 can be solved by factoring, but it requires factoring a quadratic polynomial where the leading coefficient is not 1. Our next goal is to learn about factoring this type of polynomial. We start by examining products that lead to such quadratic polynomials.

5. For each of the following, perform the indicated multiplication and use a *rectangular model* to show a geometric interpretation of the product as area for positive values of x.

a. $(2x+3)(3x+4)$ b. $(x+2)(4x+11)$ c. $(2x+1)(5x+4)$

6. For each of the following, perform the indicated multiplication (*Note: you do not need to use the rectangular model)*.

a. $(2x-3)(9x+2)$ b. $(3x-1)(x-4)$ c. $(4x-7)(2x+9)$

Factoring Polynomials

The method for factoring general quadratic polynomial of the form $ax^2 + bx + c$, with *a*, *b*, and *c* all non-zero integers, is similar to the method previously learned for factoring quadratics of this form but with the value of *a* restricted to *a* = 1. The next item guides you through an example of this method.

7. **Factor** the quadratic polynomial $6x^2 + 7x - 20$ using the following steps.

   a. Think of the polynomial as fitting the form $ax^2 + bx + c$.

   What is *a*? ____ What is *c*? ____ What is the product *ac*? ____

   b. List all possible pairs of integers such that their product is equal to the number *ac*. It may be helpful to organize your list in a table. Make sure that your integers are chosen so that their product has the same sign, positive or negative, as the number *ac* from above, and make sure that you list all of the possibilities.

Integer pair	*Integer pair*	*Integer pair*	*Integer pair*

   c. What is *b* in the quadratic polynomial given? _____ Add the integers from each pair listed in part b. Which pair adds to the value of *b* from your quadratic polynomial? We'll refer to the integers from this pair as *m* and *n*.

Integer pair, sum		*Integer pair, sum*		*Integer pair, sum*		*Integer pair, sum*	

   d. Rewrite the polynomial replacing *bx* with $mx + nx$. [*Note either m or n could be negative; the expression indicates to add the terms mx and nx including the correct sign.*]

   e. Factor the polynomial from part d by grouping.

f. Check your answer by performing the indicated multiplication in your factored polynomial. Did you get the original polynomial back?

8. Use the method outlined in the steps of item 8 to factor each of the following quadratic polynomials. Is it necessary to always list all of the integer pairs whose product is *ac*? Explain your answer.

a. $2x^2 + 3x - 54$
b. $4w^2 - 11w + 6$
c. $3t^2 - 13t - 10$
d. $8x^2 + 5x - 3$
e. $18z^2 + 17z + 4$
f. $6p^2 - 49p + 8$

9. If a quadratic polynomial can be factored in the form $(Ax+B)(Cx+D)$, where the *A, B, C,* and *D* are all integers, the method you have been using will lead to the answer, specifically called the correct ***factorization***. Show that the following quadratic polynomial cannot be factored in the form $(Ax+B)(Cx+D)$, where the *A, B, C,* and *D* are all integers. Factor the following: $4z^2 + z - 6$

Integer pair, sum		Integer pair, sum		Integer pair, sum		Integer pair, sum	

10. The method required to solve the factored quadratic polynomial is the *Zero Product Property*. Use your factorizations from item 8 as you solve the quadratic equations below (*Note: You factored these already, just make sure each equation is set equal to zero before you begin*).

a. $2x^2 + 3x - 54 = 0$

b. $4w^2 + 6 = 11w$

c. $3t^2 - 13t = 10$

d. $2x(4x+3) = 3 + x$

e. $18z^2 + 21z = 4(z-1)$

f. $8 - 13p = 6p(6-p)$

11. Now we return to our goal of solving the equation from item 4. Solve the quadratic equation using factorization and the Zero Product Property. Explain how the solution relates to the real-world situation of throwing a protein bar. Do both of your solutions make sense?

12. Suppose the cliff had been 56 feet higher. Answer the following questions for this higher cliff.
    a. What was the height of the protein bar, measured from the ground at the base of the cliff, at the instant that it left Blake's hand? What special point on the graph is associated with this information?

    b. What is the formula for the height function in this situation?

    c. If Blake wants to catch the falling protein bar, how long does he have until it hits the ground below the cliff? Justify your answer algebraically.

    d. If Blake does not catch the falling protein bar, how long it take for the protein bar to hit the ground below the cliff? Justify your answer algebraically.

13. Suppose the cliff had been 88 feet lower. Answer the following questions for this lower cliff.

    a. What was the height of the protein bar, measured from the ground at the base of the cliff, at the instant that it left Blake's hand? What special point on the graph is associated with this information?

    b. What is the formula for the height function in this situation?

    c. If Blake does not catch the falling protein bar, how long it take for the protein bar to hit the ground below the cliff? Justify your answer algebraically.

The Protein Bar Toss, Part 2

In the first part of the learning task about Blake attempting to toss a protein bar to Zoe, you found how long it took for the bar to go up and come back down to the starting height. However, there is a question we did not consider: How high above its starting point did the protein bar go before it started falling back down? We're going to explore that question now.

1. So far in Unit 5, you have examined the graphs of many different quadratic functions. Consider the functions you graphed in the first part of the Protein Bar Toss. Each of these functions has a formula that is, or can be put in, the form $y = ax^2 + bx + c$ with $a \neq 0$. When we consider such formulas with domain all real numbers, there are some similarities in the shapes of the graphs. The shape of each graph is called a *parabola*. List at least three characteristics common to the parabolas seen in these graphs.

2. The question of how high the protein bar goes before it starts to come back down is related to a special point on the graph of the function. This point is called the *vertex* of the parabola. What is special about this point?

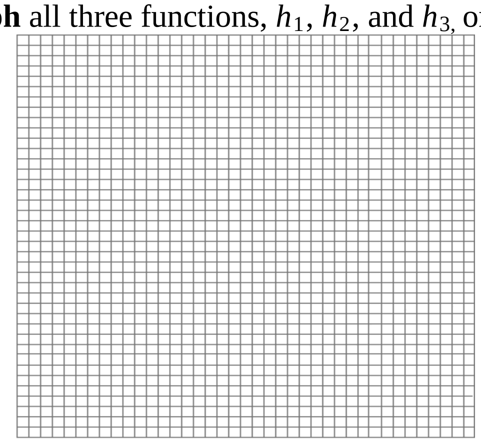
3. In the first part of the protein bar task you considered three different functions, each one corresponding to a different cliff height. Let's rename the first of these functions as h_1, so that

$$h_1(t) = -16t^2 + 24t + 160.$$

   a. Let $h_2(t)$ denote the height of the protein bar if it is thrown from a cliff that is 56 feet higher. Write the formula for the function h_2.

   b. Let $h_3(t)$ denote the height of the protein bar if it is thrown from a cliff that is 88 feet lower. Write the formula for the function h_3.

   c. **Use technology to graph** all three functions, h_1, h_2, and h_3, on the same axes.



   d. Estimate the coordinates of the vertex for each graph.

e. What number do the coordinates have in common? What is the meaning of this number in relation to the toss of the protein bar?

f. The other coordinate is different for each vertex. Explain the meaning of this number for each of the vertices.

4. Consider the formulas for h_1, h_2, and h_3.
   a. How are the formulas different?

   b. Based on your answer to part a, how are the three graphs related? Do you see this relationship in your graphs of the three functions on the same axes?

Estimating the vertex from the graph gives us an approximate answer to our original question, but an algebraic method for finding the vertex would give us an exact answer.

5. For each of the quadratic functions below, find the *y*-intercept of the graph **<u>and</u>** all other points with this value for the *y*-coordinate.

   a. $f(x) = x^2 - 4x + 9$

   c. $f(x) = -x^2 - 6x + 7$

   b. $f(x) = 4x^2 + 8x - 5$

   d. $f(x) = ax^2 + bx + c,\ \ a \neq 0$

6. One of the characteristics of a parabola graph is that the graph has a line of symmetry.
   a. For each of the parabolas considered in item 5, use what you know about the graphs of quadratic functions in general with the specific information you have about these particular functions to find an equation for the <u>line of symmetry</u>.

   b. The line of symmetry for a parabola is called the *axis of symmetry*. Explain the relationship between the axis of symmetry and the vertex of a parabola.

   c. Find the *y*-coordinate of the vertex for the quadratic functions in item 5, parts a, b, and c, and then state the vertex as a point.

   d. Describe a method for finding the vertex of the graph of any quadratic function given in the form $f(x) = ax^2 + bx + c,\ \ a \neq 0$.

7. Return to height functions h_1(t)=-16t^2+24t+160, h_2(t)=-16t^2+24t+216, and

$h_3(t)=-16t^2+24t+72$.

a. Use the method you described in item 6, part d, to find the exact coordinates of the vertex of each

b. Find the exact answer to the question: How high above its starting point did the protein bar go before it started falling back down?

8. Each part below gives a list of functions. Describe the geometric transformation of the graph of first function that results in the graph of the second, and then describe the transformation of the graph of the second that gives the graph of the third, and, where applicable, describe the transformation of the graph of the third that yields the graph of the last function in the list.

   a. $f(x)=x^2,\ f(x)=x^2+5,\ f(x)=(x-2)^2+5$

   b. $f(x)=x^2,\ f(x)=4x^2,\ f(x)=4x^2-9,\ f(x)=4(x+1)^2-9,$

   c. $f(x)=x^2,\ f(x)=-x^2,\ f(x)=-x^2+16,\ f(x)=-(x+3)^2+16$

9. Expand each of the following formulas from the vertex form $f(x)=a(x-h)^2+k$ to the standard form $\boldsymbol{f(x)=ax^2+bx+c}$.

   a. f(x)=(x - 2)^2+5 b. f(x)=4(x + 1)^2 - 9 c. f(x)= - (x + 3)^2+ 16

   d. Compare these expanded formulas to #5. How are they related?

   e. Compare the vertices in the original and expanded form. What special property do you notice?

10. For any quadratic function of the form $f(x)=a(x-h)^2+k$:

    a. What do the *h* and *k* in the formula represent relative to the function?

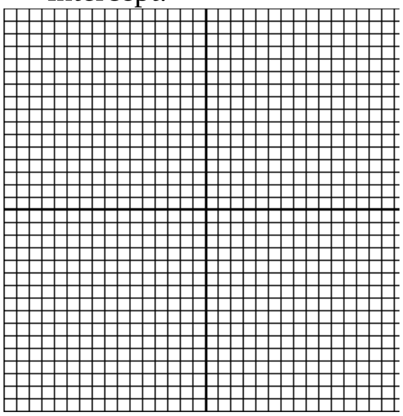
    b. Is there an alternative way to find (h, k) without finding two symmetrical points, finding the midpoint, and then finding the corresponding y value to the midpoint?

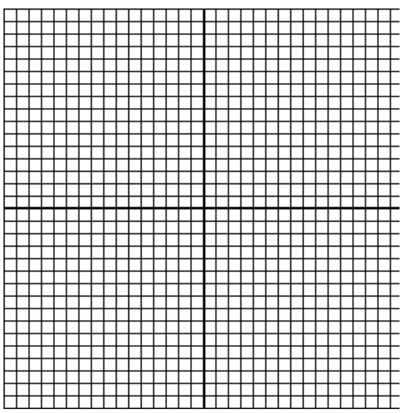
11. Use the **vertex form** of the equations for the functions h_1, h_2, and h_3 you found in #7 to verify algebraically the equivalence with the original formulas for the functions.

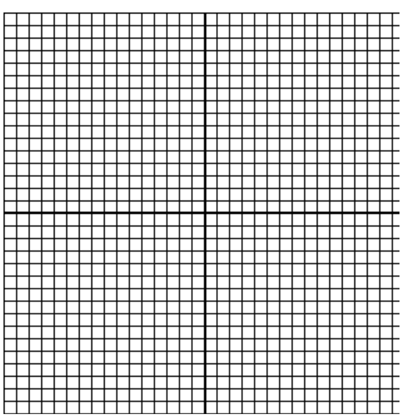
12. For the functions given below, *put the formula in the vertex* form $f(x)=a(x-h)^2+k$**,** give the *equation of the axis of symmetry*, and *describe how to transform* the graph of $y = x^2$ to create the graph of the given function.

a. $f(x) = 3x^2 + 12x + 13$ b. $f(x) = x^2 - 7x + 10$ c. $f(x) = -2x^2 + 12x - 24$

13. Make a hand-drawn sketch of the graphs of the functions in item 12. Make a dashed line for the axis of symmetry, and plot the vertex, *y*-intercept and the point symmetric with the *y*-intercept.







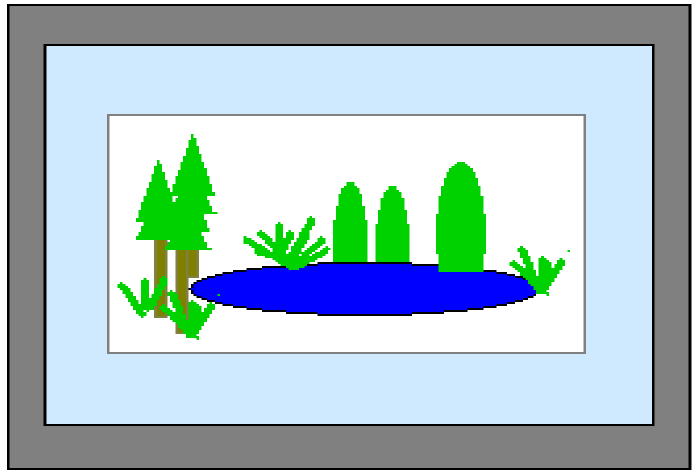
14. Which of the graphs that you drew in item 13 have x-intercepts?

    a. Find the x-intercepts that do exist by solving an appropriate equation and then add the corresponding points to your sketch(es) from item 13.

    b. Explain geometrically why some of the graphs have x-intercepts and some do not.

    c. Explain how to use the vertex form of a quadratic function to decide whether the graph of the function will or will not have x-intercepts. Explain your reasoning.

**Just The Right Border



1. Hannah is an aspiring artist who enjoys taking nature photographs with her digital camera. Her mother, Cheryl, frequently attends estate sales in search of unique decorative items. Last month Cheryl purchased an antique picture frame that she thinks would be perfect for framing one of Hannah's recent photographs. The frame is rather large, so the photo needs to be enlarged. Cheryl wants to mat the picture. One of Hannah's art books suggest that mats should be designed so that the picture takes up 50% of the area inside the frame and the mat covers the other 50%. The inside of the picture frame is 20 inches by 32 inches. Cheryl wants Hannah to enlarge, and crop if necessary, her photograph so that it can be matted with a mat of uniform width and, as recommended, take up 50% of the area inside the mat. See the image at the right.

a. Let x denote the width of the mat for the picture. Model this situation with a diagram. Write an equation in x that models this situation.

b. Put the equation from part a in the standard form $ax^2 + bx + c = 0$. Can this equation be solved by factoring (using integers)?

c. **<u>The quadratic formula</u>** can be used to solve quadratic equations that cannot be solved by factoring over the integers. Using the simplest equivalent equation in standard form, identify a, b, and c from the equation in part b and find $b^2 - 4ac$; then substitute these values in the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to find the solutions for x. Give exact answers for x and approximate the solutions to two decimal places.

d. To the nearest tenth of an inch, what should be the width of the mat and the dimensions for the photo enlargement?

2. The quadratic formula can be very useful in solving problems. Thus, it should be practiced enough to develop accuracy in using it and to allow you to commit the formula to memory. Use the quadratic formula to solve each of the following quadratic equations, even if you could solve the equation by other means. Begin by identifying *a*, *b*, and *c* and finding $b^2 - 4ac$; then substitute these values into the formula.

   a. $4z^2 + z - 6 = 0$

   b. $t^2 + 2t + 8 = 0$

   c. $3x^2 + 15x = 12$

   d. $25w^2 + 9 = 30w$

   e. $7x^2 = 10x$

   f. $\frac{t}{2} + \frac{7}{t} = 2$

   g. $3\left(2p^2 + 5\right) = 23p$

   h. $12z^2 = 90$

3. The expression $b^2 - 4ac$ in the quadratic formula is called the **discriminant** of the quadratic equation in standard form. All of the equations in item 2 had values of *a*, *b*, and *c* that are rational numbers. Answer the following questions for quadratic equations in standard form when **a, *b*, and *c* are rational numbers**. Make sure that your answers are consistent with the solutions from item 2.
   a. What kind of number is the discriminant when there are two real number solutions to a quadratic equation?

   b. What kind of number is the discriminant when the two real number solutions to a quadratic equation are rational numbers?

   c. What kind of number is the discriminant when the two real number solutions to a quadratic equation are irrational numbers?

   d. Could a quadratic equation with rational coefficients have one rational solution and one irrational solution? Explain your reasoning.

   e. What kind of number is the discriminant when there is only one real number solution? What kind of number do you get for the solution?

   f. What kind of number is the discriminant when there is no real number solution to the equation?

4. There are many ways to show why the quadratic formula always gives the solution(s) to any quadratic equation with real number coefficients. You can work through one of these by responding to the parts below. Start by assuming that each of *a*, *b*, and *c* is a real number, that $a \neq 0$, and then consider the quadratic equation $ax^2 + bx + c = 0$.
   a. Why do we assume that $a \neq 0$?

   b. Form the corresponding quadratic function, $f(x) = ax^2 + bx + c$, and put the formula for *f*(x) in vertex form, expressing *k* in the vertex form as a single rational expression.

   c. Use the vertex form to solve for *x*-intercepts of the graph and simplify the solution. Hint: Consider two cases, $a > 0$ and $a < 0$, in simplifying $\sqrt{a^2}$.

5. Use the quadratic formula to solve the following equations with real number coefficients. Approximate each real, but irrational, solution correct to hundredths.
   a. $x^2 + \sqrt{5}x + 1 = 0$
   b. $3q^2 - 5q + 2\pi = 0$
   c. $3t^2 + 11 = 2\sqrt{33}t$

d. $9w^2 = \sqrt{13}w$

6. Verify each answer for item 5 by using a graphing utility to find the *x*-intercept(s) of an appropriate quadratic function.

   a. Put the function for item 5, part c, in vertex form. Use the vertex form to find the x-intercept.

   b. Solve the equation from item 5, part d, by factoring.

7. Answer the following questions for quadratic equations in standard from where ***a*, *b*, and *c* are real numbers**.
   a. What kind of number is the discriminant when there are two real number solutions to a quadratic equation?

   b. Could a quadratic equation with real coefficients have one rational solution and one irrational solution? Explain your reasoning.

   c. What kind of number is the discriminant when there is only one real number solution?

   d. What kind of number is the discriminant when there is no real number solution to the equation?

   e. Summarize what you know about the relationship between the determinant and the solutions of a quadratic of the form $ax^2 + bx + c = 0$ where *a*, *b*, and *c* are real numbers with $a \neq 0$ into a formal statement using biconditionals.

8. A landscape designer included a cloister (a rectangular garden surrounded by a covered walkway on all four sides) in his plans for a new public park. The garden is to be 35 feet by 23 feet and the total area enclosed by the garden and walkway together is 1200 square feet. To the nearest inch, how wide does the walkway need to be?

9. In another area of the park, there will be an open rectangular area of grass surrounded by a flagstone path. If a person cuts across the grass to get from the southeast corner of the area to the northwest corner, the person will walk a distance of 15 yards. If the length of the area is 5 yards more than the width, to the nearest foot, how much distance is saved by cutting across rather than walking along the flagstone path?

10. What if you are solving a quadratic equation by using the quadratic formula, and you get a discriminant that is a negative number? Recalling your work in a previous unit, a negative square root can be written as a complex number using *i*. Solve each of the following quadratic equations using the quadratic formula and writing the answers as complex numbers.

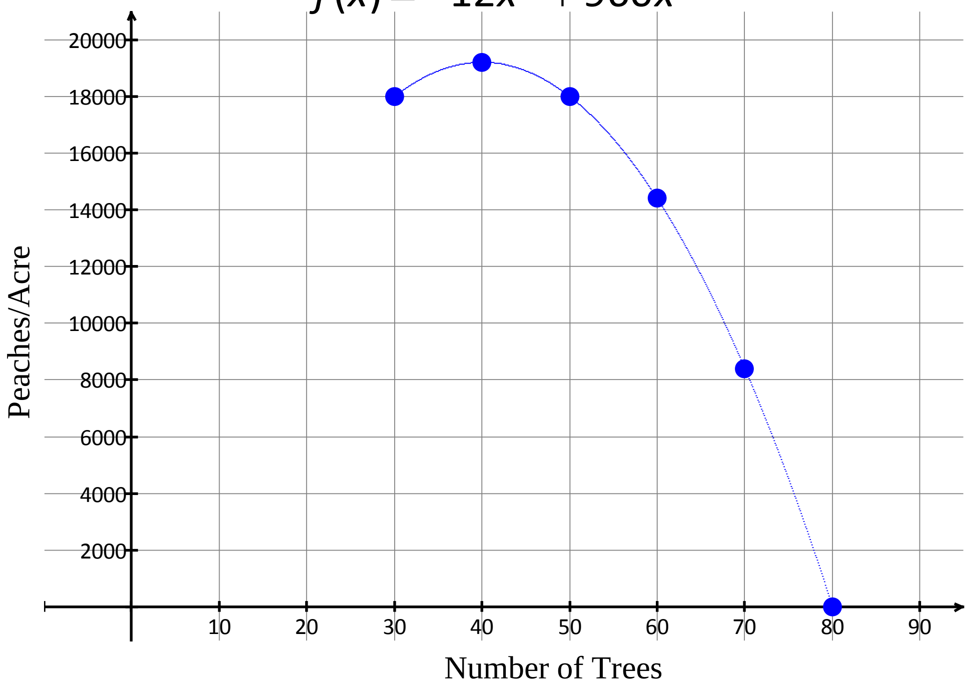
a) $x^2 + x = -6$ b) $-2x^2 + 12x - 24 = 0$ c) $5x^2 - 2x = -29$

d) $-3x^2 + x - 33 = 0$ e) $x^2 + 2x = -15$

PAULA'S PEACHES: THE SEQUEL

In this task, we revisit Paula, the peach grower who wanted to expand her peach orchard. In the established part of her orchard, there are 30 trees per acre with an average yield of 600 peaches per tree. We saw that the number of peaches per acre could be modeled with a quadratic function. The function *f(x)* where *x* represents the number of trees and *f(x)* represents the number of peaches per acre. This is given below.

$$f(x) = -12x^2 + 960x$$



1. Remember that Paula wants to average more peaches. Her current yield with 30 trees is 18,000 peaches per acre and is represented by $f(30) = 18{,}000$. (Do you see this point on the graph?)

   a. Use the function above to write an **inequality** to express the average yield of peaches per acre to be at least 18,000.

   b. Since Paula desires at least 18,000 peaches per acre, draw a horizontal line showing her goal. Does this **line** represent her goal of **at least** 18,000 peaches per acre? Why or why not?

   c. Shade the region that represents her goal of **at least** 18,000 peaches per acre.

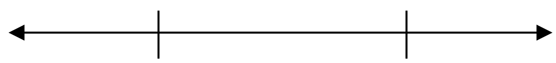
   d. Use the graph above to answer the following question: How many trees can Paula plant in order to yield at least 18,000 peaches? Write your solution as a compound inequality.

e. What is the domain of *f(x)*? In the context of Paula's Peaches, is your answer representative of the domain of *f(x)*?

2. Now let's find the answer to question 1 algebraically as opposed to graphically.
   a. Rewrite your inequality from 1a as an equation by replacing the inequality with an equal sign.

   b. The equation you just wrote is known as a **corresponding equation.** Solve the corresponding equation.

   c. When solving an inequality the solutions of the corresponding equation are called **critical values**. These values determine where the inequality is true or false. Place the critical values on the number line below. From your original inequalities, use an open circle if the value is not included and closed circle if value is included.



   d. You can now test your inequality by substituting any value from a given interval of the number line into the inequality created in 1a (original inequality). All intervals that test true are your solutions. Test each interval (left, middle, right) from your number line above. Then indicate your testing results by shading the appropriate intervals. Write your solution as a compound inequality.

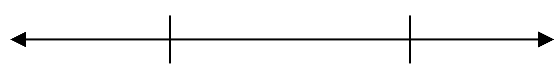
   a. Compare your test in 2d to your answer in 1d. What do you notice?

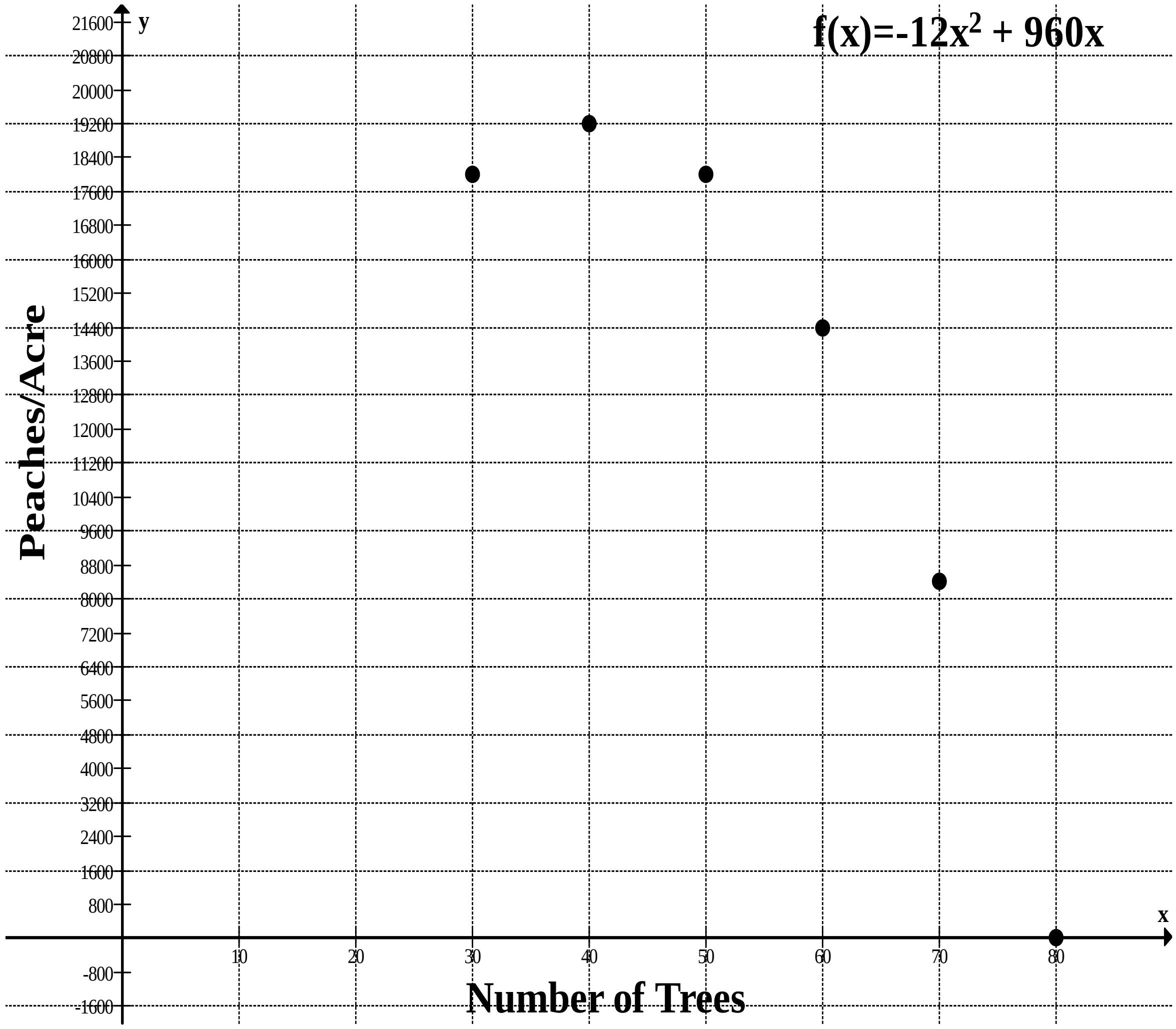
3. Paula must abide by a government regulation that states any orchard that produces more than 18,432 peaches will be taxed.

   a. Write an inequality to express when she will not be taxed.

   b. Write a *corresponding equation* and solve, finding the critical values.

   c. Now use a number line to solve the inequality as in part 2d. Write your answer as an inequality.





4. One year, a frost stunted production and the maximum possible yield was 14,400 peaches per acre.

   a. Write an inequality for this level of peach production using the function above.

   b. Since parabolas are symmetric, plot the reflective points on the graph above.

   c. Draw a horizontal line representing the maximum possible yield 14,400.

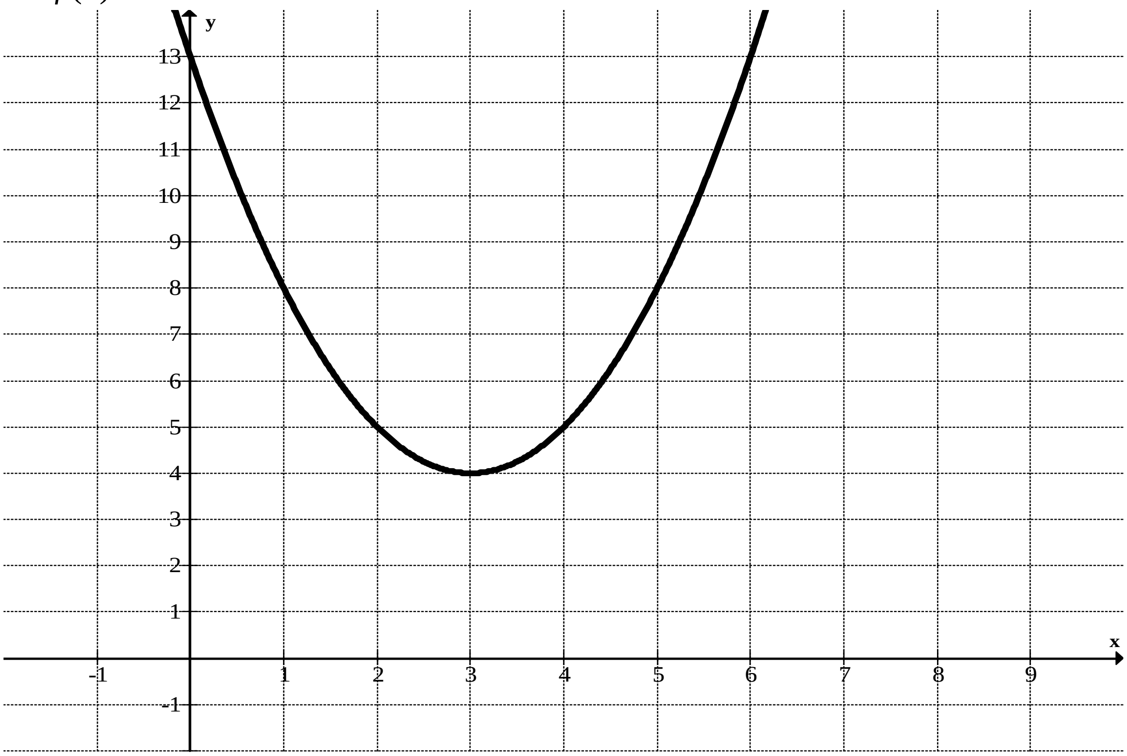
   d. Shade the region that represents her maximum yield of 14,400 peaches per acre.

e. Are any of these values not in the original domain? Explain your answer and write your final solution as an inequality.

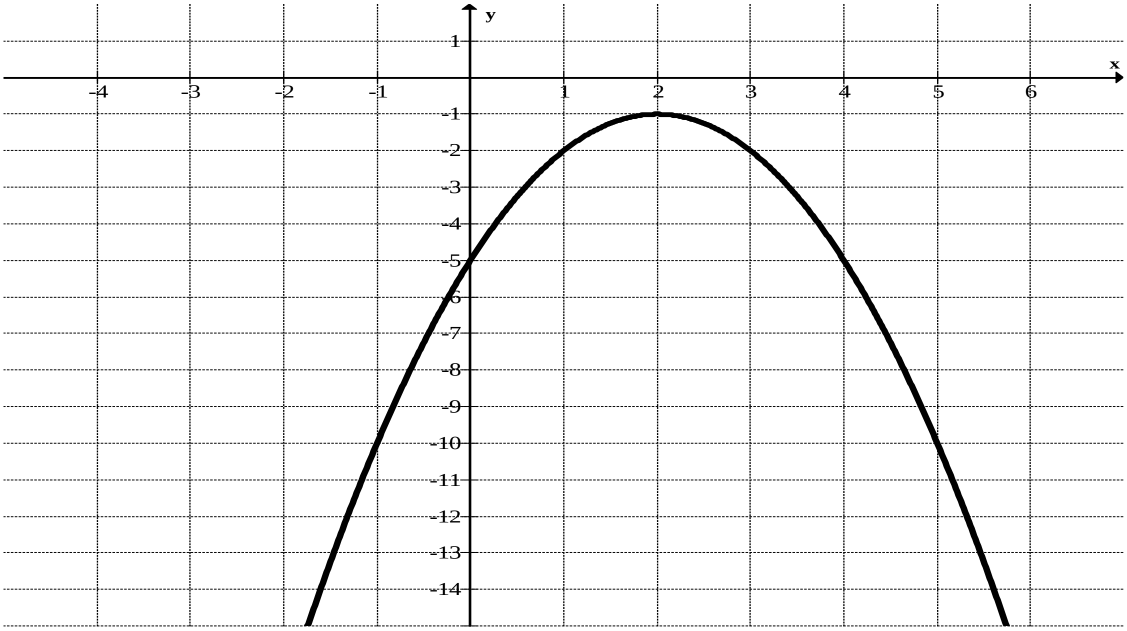
Practice Problems

For each graph below, solve the given inequality, writing your solution as an inequality. Each quadratic equation is given as $f(x)$.

1. $f(x) \geq 8$

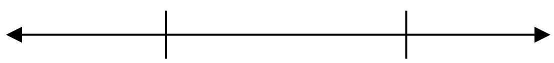


2. $f(x) \geq -10$

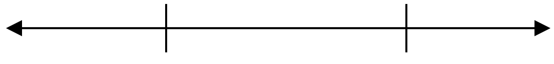


3. Solve the following inequalities algebraically.

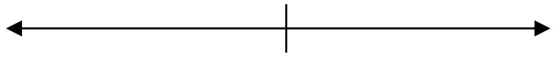
a.) $x^2 - 4x - 2 < -5$



b.) $3x^2 - 5x - 8 > 4$



c) $x^2 + 6x \geq -9$



ACME FIREWORKS

The Acme Fireworks Company has been engaged to provide the fireworks for the annual 4th of July fireworks show for the town of Madison, GA. The City Manager of Madison selected Acme Fireworks because they have four different kinds of fireworks. Each firework is designed to explode when the rocket reaches its highest point in the air. Once the rocket explodes, the sparkles of different colors fly out of the rocket and stay in the air for the same number of seconds as it took for the rocket to reach the highest point. All fireworks are launched from the ground. The time from x= 0 to the first x-intercept is the time it takes the wick to burn until the rocket is launched. The City Manager has visited your classroom to ask your class to confirm the figures that Acme Fireworks Company has given him. Specifically, he wants to know the height at which the rockets will explode and how long the sparkles for each type of firework will be in the air. Your teacher is interested in the method that you choose to solve each equation.

Below, you are given the equation for the flight of each firework. For each brand, tell the height of the rocket when it explodes, how many seconds it took to reach this height, and how long the sparkles will remain in the air. Lastly, explain why you chose the method you did to solve the given equation. In each equation, x stands for the number of seconds the rocket is in the air, and $f(x)$ models the flight of the rocket in feet.

1. Blue Bombers: $f(x) = 81 - x^2$

   i. How high is the rocket when it explodes?
   ii. How many seconds was the rocket in the air before it exploded?
   iii. How many seconds did the sparkles stay in the air?
   iv. What method did you use to solve the given equation?

2. Red Rockets: $g(x) = -2x^2 + 29x - 90$

   i. How high is the rocket when it explodes?
   ii. How many seconds was the rocket in the air before it exploded?
   iii. How many seconds did the sparkles stay in the air?
   iv. What method did you use to solve the given equation?

3. Green Gammas: $-x^2 + 12x = 15$

   i. How high is the rocket when it explodes?
   ii. How many seconds was the rocket in the air before it exploded?
   iii. How many seconds did the sparkles stay in the air?
   iv. What method did you use to solve the given equation?

4. Orange Orthognals: $h(x) = -6x^2 + 53x - 91$

   i. How high is the rocket when it explodes?
   ii. How many seconds was the rocket in the air before it exploded?
   iii. How many seconds did the sparkles stay in the air?
   iv. What method did you use to solve the given equation?

5. Acme Fireworks Company tried to design the ultimate firework called the Silver Sparkles. The equation for the Silver Sparkles was $j(x) = -x^2 + 8x - 55$. Explain why these fireworks literally never made it off the ground!

6. Now that you are officially an expert at evaluating fireworks, the Acme Fireworks Company would like for you to write an equation for their new ultimate firework called the Golden Fractals. The company wants the Golden Fractals to explode 65 feet above the ground, and they want the sparkles to stay in the air for eight seconds.

   Write an equation for the Golden Fractals. Fully explain how you got each number for the equation.

CULMINATING TASK: Quadratic Fanatic and the Case of the Foolish Function

Adapted From Jody Haynes, Fayette County School System

"Quadratic Fanatic, we need your help!" declared the voice on the answering machine. While out helping solve the town's problems, a crime had occurred at the Function Factory, and now it was up to the Quadratic Fanatic to straighten things out.

When he arrived at the factory, Quadratic Fanatic was given three different groups of suspects, each group representing a different shift. He was told that an employee from each shift had worked together to commit the crime.

The employees from the first shift were all quadratic functions in vertex form. "They are always acting kind of shifty," said the manager. The list of suspects from the first shift is below. For each suspect, list the transformational characteristics of the function.

Function	Vertical Shift	Horiz. Shift	Vertical Stretch/Shrink	Reflected?
A. f(x) = $-\frac{1}{2}(x-3)^2-4$				
B. f(x) = $2(x-4)^2+3$				
C. f(x) = $2(x+4)^2-3$				
D. f(x) = $\frac{1}{2}(x-4)^2+3$				
E. f(x) = $-2(x+4)^2+3$				
F. f(x) = $-4(x-3)^2-2$				
G. f(x) = $3(x+4)^2-2$				

According to a several witnesses, the following information about the suspect was gathered:

"He was shifted up three." Which of the employees above could be suspects?__________

"His axis of symmetry was x = -4." Which of the employees above could be suspects?___

"He had a vertical stretch of 2." Which of the employees above could be suspects?______

"I could see his reflection." Which of the employees above could be suspects?_________

Based on the above information, which employee is guilty? Explain how you know.___________

The employees from the second shift were all quadratic functions in standard form. "They always follow standard procedure," said the manager. The list of suspects from the second shift is below. For each suspect, factor and find its solutions.

Function	Factors	First Solution	Second Solution
H. g(x) = $3x^2 - 10x + 3$			
I. g(x) = $3x^2 - 21x + 30$			
J. g(x) = $2x^2 - 2x - 4$			
K. g(x) = $x^2 - x - 12$			
L. g(x) = $x^2 + 3x - 18$			
M. g(x) = $x^2 - 12x + 35$			
N. g(x) = $5(x-4)^2 - 125$ Hint: Solve by Square Root Method!			

According to a several witnesses, the following information about the suspect was gathered:

"Both solutions were integers." Which of the employees above could be suspects?______

"One of the solutions was negative." Which of the employees above could be suspects?__

"One of the solutions was two." Which of the employees above could be suspects?_____

"One of the solutions was negative one." Which of the employees above could be suspects?_____

Based on the above information, which employee is guilty? Explain how you know.__________

The employees from the third shift were also quadratic functions in standard form. "Most of them seem complex, but it could just be my imagination," said the manager. The list of suspects from the third shift is below. For each suspect, use the quadratic formula to find its solutions.

Function	Values of a, b, and c	Quadratic Formula	Solutions
O. h(x) = $2x^2 - 3x + 2$			
P. h(x) = $x^2 + 4x - 2$			
Q. h(x) = $2x^2 - 2x + 1$			
R. h(x) = $x^2 + 4x - 2$			
S. h(x) = $\frac{1}{2}x^2 - 3x + 2$			
T. h(x) = $x^2 - 6x + 10$			
U. h(x) = $x^2 + 4x - 1$			

According to a several witnesses, the following information about the suspect was gathered:

"The solutions were imaginary." Which of the employees above could be suspects?_____

"The solutions were not radical." Which of the employees above could be suspects?____

"The solutions did not involve fractions." Which of the employees above could be suspects?_____

Based on the above information, which employee is guilty? Explain how you know.___________

____

What three functions committed the crime? ____

BONUS: What mode of transportation did they use to make their getaway?______